Tensor triangular geometry and Kasparov's equivariant KK-theory

Ivo Dell'Ambrogio

Abstract

This talk presents results of my PhD thesis.

For any second countable locally compact Hausdorff group G, Gennadi Kasparov [Kas88] has defined a category KK^G with objects separable G-C^*-algebras. It is naturally endowed with the structure of a tensor triangulated category, and it plays for G-C^*-algebras roughly the same central rôle as the derived category does for modules over a ring. Notably, it is used in a quite general formulation of the famous "Baum-Connes Conjecture (with coefficients)" for the group G [BCH94].

Now, Paul Balmer [Bal05] has introduced an invariant for a general tensor triangulated category $\mathcal{T}$, called the *spectrum of* $\mathcal{T}$ and written $\operatorname{Spc}\mathcal{T}$. This is a topological space which allows for a transfer of many useful ideas from algebraic geometry to any context where tensor triangulated categories arise.

Applying Balmer's theory to results of Ralf Meyer and Ryszard Nest [MN06], I show that a strong form of the Baum-Connes Conjecture holds for G if the space $\operatorname{Spc} KK^G$ is covered by the union $\bigcup_H \operatorname{Spc} KK^H$, where H varies among all the compact subgroups of G.

If time permits, I will present the partial results I have so far obtained towards an understanding of the spaces $\operatorname{Spc} KK^G$.

References

[Bal05] Paul Balmer, *The spectrum of prime ideals in tensor triangulated categories*, J. Reine angew. Math **588**, (2005) 149–168

[BCH94] P. Baum, A. Connes, N. Higson, *Classifying space for proper actions and K-theory of group C^*-algebras.* (English Summary). C^*-algebras: 1943–1993 (San Antonio, TX, 1993), 240–291, Contemp. Math. **167**, Amer. Math. Soc., Providence, RI (1994)

[Kas88] G. G Kasparov, *Equivariant KK-theory and the Novikov conjecture*, Invent. Math. **91** (1988) no. 1, 147–201

[MN06] Ralf Meyer and Ryszard Nest, *The Baum-Connes conjecture via localization of categories*, Topology **45** (2006) 209–259