Classifications of solutions to the higher order Liouville's equation on $\mathbb{R}^{2m}$

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Abstract

Solutions to the higher order Liouville's equation on $\mathbb{R}^{2m}$

$$(-\Delta)^m u = (2m-1)!e^{2mu} \tag{1}$$

correspond to conformal metrics $g = e^{2u}g_{\mathbb{R}^{2m}}$ of constant Q-curvature $Q_g \equiv (2m-1)!$. We classify the solutions to (1) with finite total Q-curvature in terms of analytic and geometric properties. The analytic conditions involve the growth rate of u and the asymptotic behavior of $\Delta u(x)$ as $|x| \to \infty$. As a consequence we give a geometric characterization in terms of the scalar curvature of the metric $e^{2u}g_{\mathbb{R}^{2m}}$ at infinity, and we observe that the pull-back of this metric to S^{2m} via the stereographic projection can be extended to a smooth Riemannian metric if and only if it is round.