

Monotone Volume Formulas for Geometric Flows

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27.11.2007

Abstract

In this talk, we consider geometric flows where the Riemannian metric on a closed manifold M evolves according to

$$\frac{\partial}{\partial t} g_{ij}(t) = -2S_{ij}(t). \quad (1)$$

Here, S_{ij} is any symmetric tensor on M . A typical example would be the case where S_{ij} is the Ricci tensor of $(M, g(t))$ and the metric $g(t)$ is a solution to the Ricci flow. A crucial result in Perelman's analysis of the Ricci flow and his proof of the Poincaré conjecture is the monotonicity of the so-called *backwards reduced volume* under the Ricci flow. It is natural to ask whether his result has an analog for general geometric flows of the form (1). We prove that if S_{ij} satisfies a certain evolution inequality, we can construct (forwards and backwards) reduced volume quantities which are monotonically non-increasing or non-decreasing under the flow (1), respectively. With the help of this we extend Perelman's result to a large family of geometric flows.